MODELING DIAMETERS AND HEIGHTS OF IMPROVED SLASH PINE (*PINUS ELLIOTTII* ENGELM. VAR. *ELLIOTTII*) USING WEIBULL DISTRIBUTIONS

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Abstract.--To model the heights and DBH distributions of 15-year-old slash pine progenies in southeast Georgia, the two and three parameter Weibull functions were used. Both fit the data equally well, but the three parameter form, due to its more meaningful interpretation, was used to examine changes in the distributions over time, site, and among progenies. Age and site significantly influenced both the location and scale parameters of the height and DBH distributions. Across ages and sites, progeny differences were detected only in the shape parameter of the DBH distribution. However, a balanced subset of progenies and checklots suggested that the DBH distributions of progenies and checks differ in location and shape parameters. The findings reported here will be used in the development of yield functions for genetically improved slash pine.

Additional keywords: Growth and yield studies, diameter distributions, height distributions, tree improvement.

Growth and yield functions currently used in pine plantations of the Southeast are based on data gathered from genetically unimproved stands. Of the 73 slash pine growth and yield studies listed by Williston (1975) none were concerned with genetically improved stock.

Modeling diameter (DBH) and height distributions is the first step toward the development of yield formulae. Bailey and Dell (1973) list several examples of previous diameter distribution models. Predictions of these distributions can help the forester forecast the future value of a stand, estimate the number of trees which meet merchantability requirements, plan thinning operations and determine harvesting costs.

This paper discusses modeling the DBH and height distributions of half-sib progenies of 15-year-old slash pine in southeast Georgia. The estimated parameters of the distribution models are then evaluated for the effects of test (site), age, and progeny.

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MATERIALS AND METHODS

Progeny Tests

Five half-sib slash pine progeny tests maintained by Brunswick Pulp Land Company in Appling, Brantly, and Wayne Counties, Georgia, were included in this study (Table 1). The tests were planted at a 7' x 12' spacing following a site preparation with KGing, raking, harrowing, and bedding. All tests were planted in a randomized complete block design with two or three checklots replicated within each block. The tests vary in size, number of families, number of replications, type of family plot, and site index at base age 25.

Table 1.--Description of half-sib progeny tests included in this study.

Test	Location	Established	Site Index	No. of Progenies	No. of Reps	Type of Plot	Measurement Ages
1-3	Wayne Co.	1963	60	(2)[1/]	10	10-tree row	5,7,10,15
1-4	Wayne Co.	1963	65	7 (2)	10	10-tree row	5,7,10,15
1-5	Wayne Co.	1963	70	14 (2)	40	Single tree	3,5,10,15
1-6	Appling Co.	1964	65	38 (3)	5	2-tree row	7,10,15
1-7	Brantly Co.	1964	60	38 (3)	5	7-tree row	7,10,15

[1/] Number of checklots.

Statistical Analysis

The two and three parameter Weibull distribution functions were selected because of their flexibility to model tree diameters and heights (Bailey and Dell, 1973). The two parameter Weibull probability density function is of the form:

$$f(x) = \left(\frac{c}{b}\right)\left(\frac{x}{b}\right)^{c-1} e^{-\left(\frac{x}{b}\right)^{c}} \quad \text{where } x \geq o,\ b > o,\ c > o.$$

The more general three parameter Weibull probability density function is as follows:

$$f(x) = \left(\frac{c}{b}\right)\left(\frac{x-a}{b}\right)^{c-1} e^{-\left(\frac{x-a}{b}\right)^{c}}\ ;\ x \geq a \geq o,\ b > o,\ c > o.$$

In the expressions above, x represents DBH or height with the parameters a, b, and c estimated from the data. For this study the statistics for the two parameter Weibull were estimated by the maximum likelihood method (Thoman, Bain, and Antle, 1969). For the three parameter Weibull, estimates were obtained by the simple percentile procedure (Zanakis, 1979). Parameter estimates for both Weibull models were calculated using diameter or height measurements from each surviving tree of a given progeny within a given test at each measurement age. The modified Komogarov-Smirnov statistic was used to determine which model best fit a given distribution.

The three estimated Weibull parameters of each distribution were used as response variables for analysis of variance with test, age, and progeny as factors. Eleven progenies and two checklots were included in the analysis. Individual progenies were represented in at least three tests and all measurement ages with DBH distributions analysed at age 7 and above. Additional analyses were calculated for the 10- and 15-year DBH distributions using parameter estimates of the four progenies and two checklots common to all five tests.

RESULTS

Though both Weibull models fit the data equally well the three parameter Weibull was chosen for further analysis because its parameters have a more meaningful interpretation. This choice was made in spite of the fact that the maximum likelihood estimates of the two parameter Weibull are generally regarded as better estimates.

In the three parameter Weibull, "a", the location parameter, shifts the distribution along the x axis. In progeny test 1-3 the height distribution value for "a", averaged over the progenies at age 15 was 19.71 feet (Table 2). This parameter represents the smallest possible element in the distribution. The scale parameter, "b", controls the spread of the distribution with a+b defining the point at which 63% of the trees are smaller. This parameter was the greatest in the height distributions of test 1-6. The symmetry of the distribution is explained by "c". At c = 3.6 the curve is symmetrical and approximates normal. When "c" decreases, the curve becomes positively skewed with a tail increasing to the right. As "c" increases beyond 3.6 the distribution becomes negatively skewed. All the progenies in test 1-5 at age 15 had DBH distributions that were positively skewed.

Univariate analyses of variance of the three Weibull parameters for total tree heights indicate that the location parameter, "a", is significantly affected by age and test (Table 3). The scale parameter, "b" also has significant test and age components of variation with age having the greatest effect. There were no significant main effects in the shape parameter, "c". The test by age and the test by progeny interactions were significant in all the parameters of the height distributions. Figure la shows the changes in the height distribution of one progeny in the single-tree plot test through time. The effects of tests are shown in Figure lb for one progeny's height distribution on a poor site (Test 1-3) and a good site (Test 1-5).

Table 2.--Average and range of progeny values for estimated Weibull parameters for the height and DBH distributions at age 15.

Test	Height Distribution Parameter a	Height Distribution Parameter b	Height Distribution Parameter c	DBH Distribution Parameter a	DBH Distribution Parameter b	DBH Distribution Parameter c
1-3	19.71[1]	20.93	4.04	2.75	3.09	3.32
	10.94-25.19[2]	14.50-31.56	2.24-6.81	1.47-3.47	2.20-4.56	4.56-5.49
1-4	23.19	21.38	4.88	2.89	3.54	3.63
	5.29-34.00	10.30-39.71	2.54-8.04	2.01-4.30	2.20-4.39	2.03-5.05
1-5	43.52	11.70	2.63	5.28	2.70	2.42
	39.75-49.92	7.08-15.64	1.17-4.51	3.52-6.10	2.00-4.58	1.51-3.33
1-6	15.09	29.88	4.30	2.02	4.32	3.31
	0.00-28.47	15.53-46.99	1.84-7.77	0.00-3.67	2.43-6.65	1.51-5.91
1-7	25.29	27.68	4.85	2.97	4.20	3.26
	0.00-41.91	9.09-51.99	1.03-10.69	0.00-4.24	3.01-7.50	1.95-7.86

1/ Mean.
2/ Range.

Table 3.--Analysis of variance of the 3 Weibull parameters of the height and DBH distributions.

Source	Height Distribution df	a		c	DBH Distribution df	a		c
Tests (T)	4	21.45**	6.75**	1.60	4	17.04**	2.89*	0.65
Ages (A)	4	19.33**	13.85**	2.61	2	9.54*	19.34**	2.64
Progenies (P)	4	0.85	0.88	1.56	12	0.82	1.12	2.09*
T x A	9	4.99**	3.05**	2.39*	7	2.40*	3.34**	1.93*
T x P	36	1.95**	1.84*	1.77*	36	2.00**	2.01**	1.84*
A x P	48	0.85	0.89	1.28	24	1.37	1.52*	1.28

* and ** Significant at 5 and 1% levels, respectively.

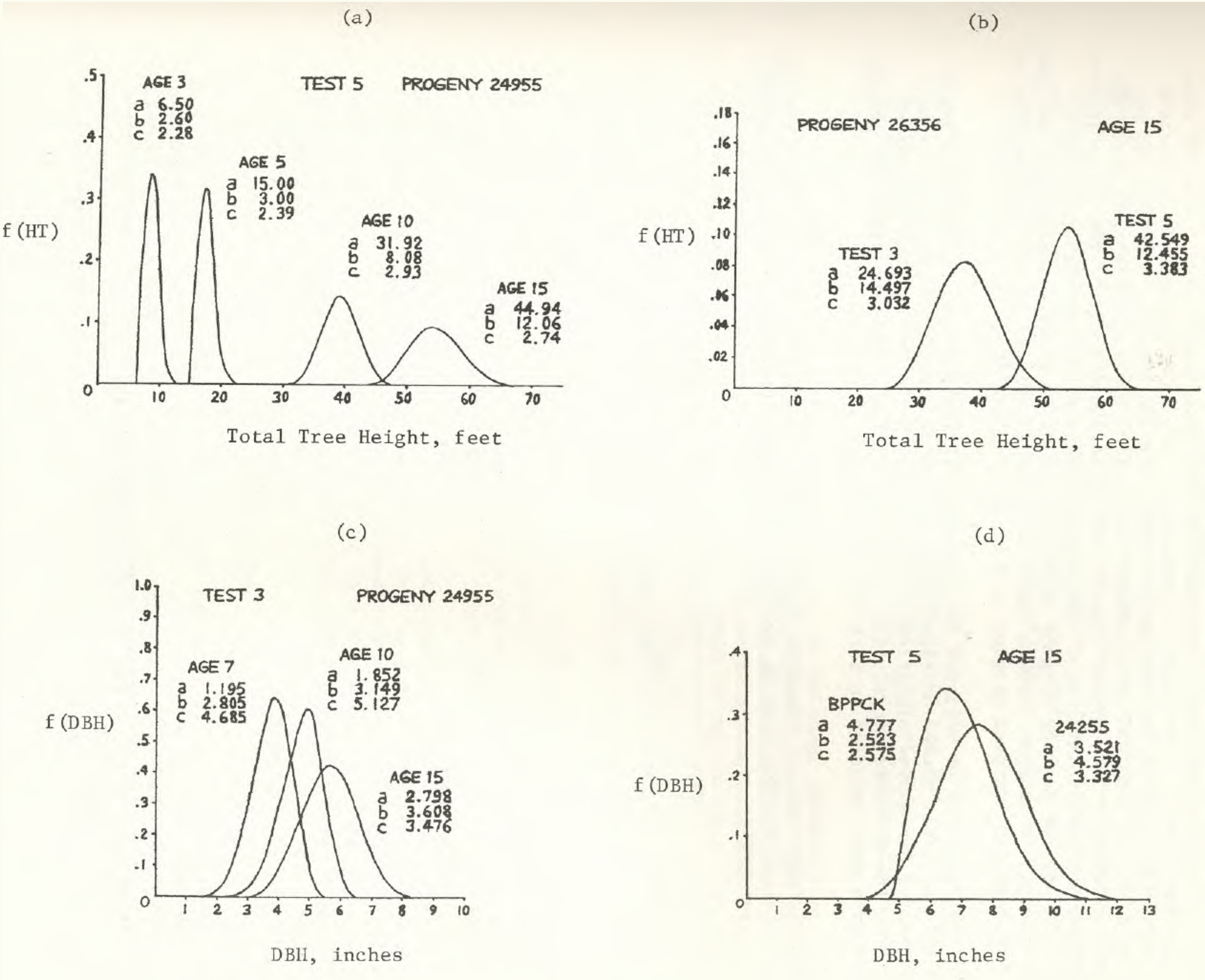

Figure 1. Influence of various factors on height and DBH distribution: a) age on height, b) site on height, c) age on DBH, d) progeny on DBH.

Univariate analyses of variance for the three parameters of the DBH distribution show that the location parameter, "a", is significantly influenced by age, test and the interactions between test and progeny, and test and age (Table 3). This parameter is greatly controlled by test (site) and age. The scale parameter, "b", showed significant effects due to test, the three interactions and particularly age. The shape parameter has significant progeny, test by age and test by progeny effects. Figure lc gives an example of the DBH distribution's changes through time. Also shown (Figure 1d) is a comparison of a check and an improved progeny in test 1-4 at age 15.

The analysis of variance for the DBH distributions of the progeny common to all the tests showed no significant parameter differences between the two checklots, but showed significant differences in the "b" and "c" parameters between the improved progeny (Table 4). Significant check-vs.-improved progeny contrasts were detected in both the "a" and "c" parameters.

Table 4.--Analysis of variance of the 3 Weibull parameters of the DBH distributions over ages 10 and 15, and of the 6 progenies common to all five tests.

Source	df	a	b	c
Test (T)	4	10.14**	1.57	2.34*
Age (A)	1	26.09**	23.15**	3.09
Progenies (P)				
Checks	1	0.22	0.03	0.00
Improved	3	1.33	3.56*	5.50*
Checks-vs.-Improved	1	4.39*	1.57	11.35**
T x A	4	1.18	1.26	0.68
T x P				
Checks	4	3.74	3.21	6.14
Improved	12	5.64**	4.50**	1.25
Checks-vs.-Improved	4	3.99	4.07	8.53**
A x P				
Checks	1	3.17	2.81	10.28*
Improved	3	1.19	1.38	0.20
Checks-vs.-Improved	1	0.00	0.01	0.03

* and ** Significant at 5 and 1% levels, respectively.

DISCUSSION

As expected, the age of the plantations greatly affected the DBH and height distributions. When trees are planted, they have approximately uniform size. The DBH and height distributions of a very young stand have very little spread and appear as a spike over some low value. As age increases, diameter and height distributions shift to the right, as indicated by the larger location parameter, "a". With time, the spread of the distribution increases, changing the scale parameter, "b".

The effects of the test, which should reflect site effects, are greatest in the location parameter. Site affects tree height and DBH, with some effect on the spread of the distribution, and no significant effect on the shape. The significant test-age interaction is probably due to faster growth on better sites.

The progeny effect was only significant in the shape parameter of the DBH distribution. The analysis of the six progenies common to all five tests over ages 10 and 15 showed that the improved progenies' DBH distributions had significant differences in shape, but most of the shape parameter's significance was due to the check-vs.-progeny contrast. This second analysis also showed significance in the checks-vs.-progeny contrast of the location parameter "a". These two significant contrasts suggest that the improved progenies have a greater proportion of larger diameter trees than the checklots. Also the test by progeny interaction indicates that progenies perform differently in different tests.

The estimation procedure used here to calculate the three Weibull statistics was very sensitive to outlying trees, especially very small trees. This may be the reason for the shape parameter of the DBH distribution being the only significant progeny effect. Other estimation techniques may produce different results.

Knowing the effects of age, test, and genetics on the DBH and height distributions will be extremely helpful in the further development of growth and yield formulae. The next step toward these yield functions will be the formulation of multiple regression equations to predict values of the distribution parameters as functions of age, site index, planting density and some measure of the genetic component.

CONCLUSIONS

For our data, progeny test and stand age have significant effects on the estimated "a" and "b" Weibull parameters in both the height and DBH distributions. Progeny was only significant for the shape of "c" parameter of the DBH distribution. The parameter estimation procedure used was very sensitive to outlying trees. A method of screening the data to remove these outliers should reduce the error terms and result in more significant progeny effects.

LITERATURE CITED

Bailey, R. L. and T. R. Dell. 1973. Quantifying diameter distributions with the Weibull function. Forest Solo 19:97-104.

Thoman, 0. R., L. J. Bain, and C. E. Antle. 1969. Inferences on the parameters of the Weibull distribution. Technometrics 11:445-460.

Williston, H. L. 1975. Selected bibliography on growth and yield of the four major southern pines. U.S.D.A. Forest Service, Southeastern Area, State and Private Forestry. 27p.

Zanakis, S. H. 1979. A simulation study of some simple estimators for the three-parameter Weibull distribution. J. Statist. Comput. Simul. 9: 101-116.