RCDIob: An Individual Tree Growth and Yield Model for LobIolly Pine That Incorporates Root-Collar Diameter at Time-Of-Planting

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Introduction

There are many growth and yield models for loblolly pine (Pinus taeda L.) plantations in the US (Smalley and Bailey 1974, Baldwin and Feduccia 1987, Lenhart 1996, Baldwin and Cao 1999, Burkhart et al. 2004). However, none relate root-collar diameter (RCD) at the time-of-planting to future tree growth. In fact, throughout the world, very few models have related seedling size at establishment to future growth and yield. Ground-line diameter (GLD) at time-ofplanting was used in New Zealand as a predictor variable to estimate tree growth and survival of Pinus radiata D. Don (Mason 2001). In South Africa, survival up to two years after planting was modeled for P. radiata (Zwolinski et al. 1994). Although GLD at time-of-planting has a strong relationship with RCD (VanderSchaaf and South 2003), a nursery manager might be mislead into producing a smaller than intended target RCD when given an output from a model that is based on initial GLD. Factors such as planting depth, top pruning effects on stem taper, and perhaps genetic differences in taper between families of loblolly pine seedlings can produce variability in the relationship between RCD and GLD at time-of-planting.

Nurseries in the Southeastern US usually produce seedlings with an average root-collar diameter (MRCD) at planting of less than 4 mm (South et al. 2001). Such small seedlings can be produced inexpensively since when grown close together, the costs associated with herbicides, fertilization, lifting, etc. are minimized. Since planters are paid based on the number of seedlings planted and not on the number of seedlings surviving after a certain amount of time, crews prefer to plant seedlings with small root systems (South et al. 2001). Growing larger seedlings at lower seedbed densities will increase the cost of production (by perhaps \$4-7 per thousand seedlings). Larger seedlings will also reduce hand-planter revenue per hour because of greater seedling root mass that requires more time to plant properly (Caulfield et al. 1987, South et al. 2001). However, seedling root mass does not impact the speed of machine-planting. Wider spaced, morphologically-improved seedlings (MI) generally cost more to produce and purchase than standard seedlings (South et al. 2005) but they have exhibited greater growth

and survival compared to standard seedlings (Shoulders 1961, Shipman 1964, Sluder 1979, South et al. 1985, Dierauf et al. 1993, South 1993, South et al. 1995, South et al. 2001, South and Rakestraw 2002). Planting loblolly pine seedlings that were grown at low nursery densities may provide economic advantages (Caulfield et al. 1987, South 1993, South and Rakestraw 2002, South et al. 2005), especially when outplanted at wider spacings (South 1993, South and Rakestraw 2002, South et al. 2005).

Resource managers in the US need tools to assist in determining the economic trade-offs between (1) increased seedling and outplanting costs, and (2) greater survival and growth from planting MI seedlings (plus lower per unit area establishment costs due to a reduction in the number of seedlings outplanted). Currently, to make such calculations, a manager must make assumptions about the performance of MI seedlings. Although published papers provide information to support such assumptions, a predictive model would allow resource managers to conduct growth and economic analyses using costs and revenues that are more specific to their particular area. Additionally, a modern establishment model would allow users to vary the distribution of seedling sizes at time-of-planting (e.g. Mason 2001). The objective of this research was to develop individual tree mortality, diameter at breast height (DBH), and height models to relate RCD at time-of-planting to future growth and economic returns.

Methods

We used a distance-independent model procedure to relate growth to RCD at time-of-planting. A logistic model was used to estimate the probability of individual tree survival (Hamilton 1986, Flewelling and Monserud 2002, Moore et al. 2004).

Data

Data were obtained from four sites located on the Atlantic Lower Coastal Plain in Georgia and South Carolina (South et al. 2001, VanderSchaaf and South 2003). Two seedling ideotypes (average across all sites of 5.0 mm for the standard seedlings and 8.5 mm for the morphologicallyimproved seedlings) were planted. Two plantation regeneration management scenarios (1 – standard, 2 - intensive) were used. Both scenarios involved raking, piling, and burning all residual debris followed by a bedding treatment in the summer. In addition to these site preparation treatments, the standard management scenario included a broadcast herbicide treatment of hexazinone and sulfometuron in March plus fertilization with diammonium phosphate (DAP). In addition to the standard management scenario treatment applications, the intensively managed plots received a broadcast herbicide application of imazapyr and metsulfuron in mid-summer of the planting year and again one-year later. A treatment of DAP plus potassium chloride was applied 2 years after planting. More detailed descriptions of treatments were provided by South and others (2001). Sampling age was up to 12 years old (Table 1). Planting density varied by site from 1495 to 1794/ha.

Modeling procedures

Height and diameter equations

Both linear and nonlinear regression equations were examined for height and DBH. Equations for a particular dependent variable were selected using both statistical and biological properties but biological properties were the overriding concern. Parameters were checked to make sure that they were consistent with biological theory (e.g. greater numbers of trees per ha surviving at a particular age should not increase the estimate of DBH). Additionally, predicted values of stand development were examined to determine whether a particular equation or sets of equations produced reasonable values across a range of ages and values of the regressors. After accounting for biologically meaningful variables, we selected the function with the lowest untransformed average absolute value residual. Residual errors were examined for trends.

Since we wanted to extrapolate predictions of individual tree height and DBH beyond the range of ages in the data for economic rotation purposes, the Chapman-Richards equation was selected to model both individual tree height, [1], and DBH [2]:

Proc Model (SAS 1989) using the Gauss-Newton algorithm was used to estimate parameters in the individual tree height and DBH model functions. All variables were significant at alpha levels less than 0.0001. These models were developed using longitudinal data and thus errors are most likely correlated which can result in biased confidence intervals and hypothesis tests. However, parameter estimates are still asymptotically unbiased (Schabenberger and Pierce 2002). Since all variables were significant at alpha levels less than 0.0001 (and we were more concerned with biological meaning than statistical significance of coefficients), we ignored the correlation value when estimating parameters.

A sigmoid growth curve ensures reasonable estimates of the response variables at ages of 15, 20, 25 yr. Zeide (1989) reported the Chapman-Richards equation was superior to other sigmoid growth equations for predicting DBH. We tested other growth equations such as the Gompertz, Logistic, Monomolecular, and the Weibull but found they overestimated DBH. Zeide (1989) found the Power Decline I, or the Korf (Zeide 1993), sigmoid growth equation was superior to the Chapman-Richards equation for predicting DBH. We found the Korf equation resulted in reasonable estimates of DBH at older ages, but the combination of this equation with the mortality model, [5], resulted in an over-prediction of mortality at older ages. Thus, we used the Chapman-Richards equation to predict DBH.

The allometric relationship between individual tree height and DBH is widely known – often referred to as the constant-stress theory (Zeide and VanderSchaaf 2002). Although most use DBH to predict height, we chose to model DBH as a function of height. Our reasoning for this is because we are predicting growth at young ages (i.e. 1 and 2 years) as well as for older ages. We feel that resource managers want to know when basal area growth begins on trees in relation to RCD. One approach is to first predict height, and once predicted height values have reached DBH (4.5 feet), predict DBH. Thus, natural resource managers can use our model for not only long-

[1] Ht = 31.89971 (1- $e^{-0.06266Age}$)^{1.637244 - 0.01852RCD - 0.17411Treat}

Where: Ht – total individual tree height (in meters), Treat – 1 – Standard regeneration scenario, 2 – intensive regeneration scenario, n = 16344, Adj. R^2 = 0.9454, RMSE = 1.1259, Durbin-Watson test statistic (DW) = 1.0973

[2] DBH = [41.90266+9.146058LnHt-6.72958LnTPH](1-e^{-0.799929Age})^{19.46781-1.98092Treat}

Where: Ln – natural logarithm, DBH – individual tree DBH in cm, TPH – trees per ha, n = 12669, Adj. R^2 = 0.9125, RMSE = 1.8090, DW = 1.4876

term growth and yield and economic analyses in relation to RCD, but also to get a reasonable idea of when basal area production begins on individual trees in relation to RCD.

Quite often cross-equation correlation exists between model errors in a biological system of equations (Amateis et al. 1984, Borders 1989, Hasenauer et al. 1998). This is particularly true if a predicted dependent variable is an independent variable in another equation (thus the variable is an endogenous variable). Cross-equation correlation is thought to produce inefficient and possibly inconsistent parameter estimates (Borders 1989, Hasenauer et al. 1998). Hasenauer et al. (1998) state the gain in parameter efficiency is higher when equation error structures have greater correlations. If crossequation correlation exists, then when total tree height is overpredicted we would expect DBH to be overpredicted (Borders 1989). If there is a high degree of correlation between equation error structures, a Two Stage Least-Squares or Three Stage Least-Squares analysis could be used to estimate parameters. However, both Two and Three Stage Least-Squares use predetermined variables (those regressors that are not dependent variables in the equation system - thus they are exogenous or considered to be fixed) in the first stage of parameter estimation (Amateis et al. 1984, Borders 1989, Hasenauer et al. 1998) and the modeler must declare what predetermined variables are used. Amateis et al. (1984), Borders (1989), and Hasenauer et al. (1998) state the first stage uses predetermined variables to predict the endogenous variable and that these estimated endogenous variable values are used as replacements (instruments) for the observed endogenous variable values - thus Two and Three Stage Least-Squares ignore the biological model form of the endogenous variable in the first stage. Therefore, the proper selection of predetermined variables is essential - choosing non-meaningful predetermined variables could introduce more problems than if the simultaneous nature of the model system was ignored altogether. We do not want to ignore the biological model form of [1] and thus we choose not to use Two or Three Stage Least-Squares. Parameters were estimated using an alternative parameter estimation methodology for simultaneous systems presented in Borders (1989); however, the estimated parameters produced biologically incorrect estimates of DBH (it should be noted that for this particular analysis, the Ln transformation of height was not used in [2] rather untransformed height). Thus, we estimated parameters for both [1] and [2] in a recursive manner, obtained predicted values, calculated errors, and then determined the correlation between the error structures of the two models ([1] and [2]). The

cross-equation correlation was near 0.15. Due to the low cross-equation correlation, thus producing little gains if a simultaneous equation system was used (Hasenauer et al. 1998), biologically poor parameter estimation using the system presented by Borders (1989), and potential problems with using Two and Three Stage Least-Squares, we decided to treat the system as recursive.

When using [2] to predict DBH at ages of 2, 3, and 4, some illogical predictions occurred. For example, the quadratic mean diameter would decrease from age 2 to 3. Thus, we developed equations to predict diameter growth at young ages using data up to age 4. Since we recommend users not to use our model system for planting densities greater than 2400/ha, it is a reasonable assumption that DBH is independent of planting density until age 4. Once again, we assumed a recursive nature between the DBH and total tree height [1] equations:

Model [3] can be used to estimate DBH until age 4.

Mortality equations

Parameters in the mortality equations were obtained using Proc Logistic (SAS 1989). The dependent variable for [4] and [5] is the probability that a tree will survive into the next growing season. To estimate whether a tree that has not reached breast height will survive to the next growing season, [4] should be used:

$$\ln\left[\frac{P_{i}}{1-P_{i}}\right] = 4.9499 + 0.7307 \left[\frac{RCD}{MRCD}\right] \text{Treat}$$
[4]

Where: n = 1938

Once a tree reaches breast height, [5] can then be used to estimate the probability of a tree surviving into the next year:

$$\ln\left[\frac{P_{i}}{1-P_{i}}\right] = 6.8866 + 0.4621 \text{ DBH-}0.5845 \text{Age-}0.0631 \text{BA}$$
[5]

Where: BA – basal area in square meters per ha, n = 12437

Maximum-Size Density Relationships

Since we are extrapolating growth beyond age 12 yr, the model uses Maximum-Size Density Relationships (MSDRs) to constrain growth (Maguire et al. 1990, Mack and Burk 2002, Monserud et al. 2004). We recommend a Maximum Stand Density Index (MSDI) value of 1112 along with an exponent of 1.6 for equation [6]. Reineke (1933) originally estimated the MSDR boundary slope to be -1.6 and determined a MSDI of 1112 (450 x 2.47) for naturally-regenerated stands in the Western Gulf region of the US. Equation [6] was used to calculate a SDI value for a given stand age and structure.

$$SDI = TPH \left[\frac{QMD}{25.4} \right]^{1.6}$$
[6]

Previous studies have determined that a MSDI of 1112 is applicable for loblolly pine plantations in the southern US (Dean and Baldwin 1993, Hasenauer et al. 1994). A brief explanation of how MSDI constrains stand development follows. Quadratic mean diameter (QMD) and tree per ha (TPH) are predicted using a combination of equations [1], [2], [3], [4], and [5]. Only when the predicted SDI exceeds 1112 will the MSDI tool be initiated. If a stand's SDI is predicted to exceed 1112 then the estimate of QMD will be maintained but the estimate of TPH will be reduced such that a MSDI of 1112 is achieved. In order to obtain the required TPH to satisfy [6], we recommend users select those trees with the smallest DBH to die. In general, the MSDI tool is activated only for planting densities near 2470/ha.

Results and Discussion

All models have biologically meaningful parameter estimates ([1], [2], [3], [4], and [5]). Biologically correct parameter estimates aid in predicting response variables beyond the range of the regressor values used to estimate parameters. As with any model, we want to verify predicted stand development across a range of regressor values.

Verification Results

Due to the time and costs associated with measuring RCD prior to planting, it is difficult to verify our model using datasets independent of those used in parameter estimation since few independent datasets exist. Thus, we decided to verify our model by comparing our estimates of stand development to other growth and yield models. It is not the intention of the verification analysis to quantify differences. Quantifying differences between our model and other models would be difficult due to the stochastic nature of our model. Rather, we merely want to see whether our predictions are reasonable; especially for the planting densities near 740 and 2470/ha.

One main component of any empirical growth and yield model is an estimate of site productivity. In general, regardless of RCD, planting density, and regeneration scenario, estimated average height of dominants and codominants using our model is around 22.5 to 25 meters at age 25 yr. To be conservative, we used 24.4 meters as the site index value (base age 25 yr) for all models during the verification. The growth and yield models for cutover sites were; Ptaeda 3.1 (Burkhart et al. 2004), and a model developed by Baldwin and Feduccia (1987) specifically for Western Gulf loblolly pine plantations. Thus, predictions from the model by Balwin and Feduccia might not be comparable since RCDlob was fitted using data from genetically improved seedlings in the Lower Atlantic Coastal Plain. Ptaeda 3.1 is a distancedependent individual tree model while the other model is a stand-level diameter distribution model. Model structure differences between our model and the verification models should have minimal impact on growth and yield predictions.

For verification, we compared our predicted values for planting densities of 740, 1729, and 2470/ha using an RCD of 4.5 mm for all seedlings and a standard regeneration scenario up to age 25 (Fig. 1). Due to the stochastic nature of the mortality models in RCDlob, we compare predictions of 3 different runs from our model

Table 1. Summary of the data used in model fitting. Where: MI = morphologically improved seedlings, S = standard seedlings, BAH = square meters of basal area per ha, SDI = stand density index (from equation 6), Ht = arithmetic mean height, QMD = quadratic mean diameter.

Size	Age yr	BAH m²/ha	SDI	Ht m	QMD cm
MI	1			0.8	
MI	2			2.3	
MI	3	0.7	33.9	3.9	2.3
MI	4	1.2	53.7	5.4	3.0
MI	8	24.9	619.0	10.8	14.1
MI	10	28.9	698.7	12.1	15.2
MI	12	35.2	817.8	14.7	16.8
S	1			0.6	
S	2			2.0	
S	3	0.5	26.8	3.5	2.0
S	4	1.0	45.0	4.9	2.8
S	8	23.2	579.3	10.2	13.9
S	10	27.6	666.7	11.7	15.2
S	12	33.3	775.6	14.0	16.8



Figure 1. Comparison of growth projections from three RCDlob simulations (lines with no points) to two other cutover loblolly pine plantation growth and yield models (Baldwin and Feduccia 1987 – filled black diamonds, Burkhart et al. 2004 – nonfilled circles) for three different planting densities of 740 seedlings per ha (SPH), 1729 SPH, and 2470 SPH. In RCDlob, a standard regeneration scenario was selected using 4.5 mm diameter seedlings.

to the verification growth and yield model programs. Generally, data used to fit both verification models are from stands operationally planted prior to 1985. Thus, it is a reasonable assumption that the plots were planted using seedlings that had an average RCD near 4.5 mm and, at the maximum, a Standard regeneration scenario. In addition to the limits imposed by equation [6], estimates from RCDlob are based on a maximum annual diameter growth of 2.54 cm (up to age 10 yr) and a maximum annual diameter growth of 1.27 cm for older stands.

As seen in Fig. 1, and based on the verification growth and yield model program projections (Baldwin and Feduccia 1987, Burkhart et al. 2004), our model gives reliable estimates of stand development across the range of planting densities from 740 to 2470/ha. In general, our model has greater predicted survival than the two other models for young stand ages. This may be reflective of the relatively high regeneration intensities in our dataset; even for the standard regeneration scenario. In addition, our model has lower early basal area and DBH development which is consistent with our data. The model by Baldwin and Feduccia (1987) does not have an inflection point. Although the model was fit using data from plantations younger than age 10 yr, the majority of their data are from plantations older than age 10 yr and thus their model may not be highly applicable for ages younger than 5 to 10 yr.

Conclusions

Our growth and yield model is the first that allows the user to vary initial RCD for loblolly pine. In addition, we allow the user to input cost data and price data for pulpwood, chip-n-saw and sawtimber sized products. Since the model outputs volumes and net-present value economics, plantation managers can easily calculate the cost/benefit ratio for planting morphologically improved seedlings. In the past, many land managers have deferred to the hand-planter when defining the desired seedling size for outplanting. Now managers in the lower-coastal plain can determine for themselves how much revenue they are giving up by planting small-diameter loblolly pine seedlings.

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